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Adaptive Bounded Control for an Uncertain Robotic Manipulator

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Abstract-In this paper, we investigate the adaptive bounded control problem of uncertain robotic manipulator. Smooth saturation function and smooth projection operator are combined in the control design to prevent the input violation and to estimate the uncertain parameters. Then, adaptive control with predetermined bounds is design by the Lyapunov's direct method. In the result, the designed control will never reach the input saturations and the tracking error converges to the origin. Simulation results are provided to illustrate the effectiveness of the proposed approaches.

Index Terms-Robotic Manipulator, Adaptive Control, Uncertain Parameter, Input Saturation, Trajectory Tracking.

1. INTRODUCTION

Constraints appear in the most mechanical systems in real-life. Violation of constraints during operation may result undesired oscillation or system damage. Especially, for the robotic systems, the dynamics depend on a number of parameters which are unknown to the system researcher. Handling of both constraints and uncertain parameters of robots is a challenging task and has attracted much attention from researchers for their potential applications in practice.

From the above problem, we firstly need to handle the uncertainties of the robots. To design the control in such cases, the adaptive control approach can be used. The formers [1] shown that the dynamics of robotic manipulator can be parameterized by the multiplication of a regressor matrix to an uncertain parameter vector. Based on this property, several adaptive control laws have been developed such that adaptive inverse dynamics control [2], adaptive passivity-based control [3] and intermediate between the two approaches [4,5].

However, the situation will be changed when constraints are taken into account. The reason is that the constraints may destroy the stability of the system even when the free control ensures the stability of the robot. In particular, when the constraints have the form of input saturation, a combination of the Lyapunov's direct method and saturation function is a good choice to prevent the constrain violation in the control design. The excellent work used the nested saturation function to deal with the multiple integrators with bounded controls firstly was proposed in [6]. Based on this word, several bounded controllers have been developed to deal with input saturations [7]. Later, the problem is solved with the use of the smooth saturation function [8]. However, when both constraints and uncertain parameters are taken into account, the considered problem becomes more challenging. The reason is that we not only need to handle the uncertainties, but also simultaneously deal with constraints.

Motivated by the above consideration, in this paper we propose the control schema based on the computation of the control inputs with predetermined bounds. The smooth projection operator is used to ensure that the estimated parameters are always belong to a known set. Smooth saturation function and smooth projection operator are combined in the control design to prevent the input violation and to estimate the uncertain parameters. This combination allows us to design the adaptive control with predetermined bounds based on the Lyapunov's direct method. Furthermore, this combination still has effectiveness in the computation the predetermined bounds. Our design ensures that inputs will never violate the saturations while the tracking error asymptotically converges to the origin.

The rest of the paper is organized as follows. In Section 2, the control problem of the uncertain robotic manipulator with the input saturations is formulated. Section 3 presents the control design. Simulation results are provided to illustrate the effectiveness of the proposed controls in Section 4. The paper concludes in Section 5.

Notation: Throughout this paper, $\mathbb{R}^n$ denotes the Euclidean space with $n-$dimension. $\|\cdot\|$is the Euclidean norm of vector$\cdot$. $\lambda_{\min}(\cdot)(\lambda_{\max}(\cdot))$is the minimum (maximum) eigenvalue of the matrix $\cdot$. For integer indices $i$and j, $\sigma_i = [\sigma_{i1}, \sigma_{i2}, \cdots, \sigma_{in}]^T \in \mathbb{R}^n$denotes the saturation function vector with elements $\sigma_{ij}, j = 1, \cdots, n$. $A \in \mathbb{R}^{n\times n}$denotes the$n \times n-$matrix. Let two vectors$x, y \in \mathbb{R}^n$, then $x < y$ ($x \leq y$) denotes that $x_i < y_i$ ($x_i \leq y_i$).

2. PROBLEM FORMULATION

In this paper, we study the robotic manipulator described by the following equations [13]:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u \quad (1)$$

$$|u_i| \le \bar{u}_i,\ i = 1, \cdots, n \quad (2)$$

where $q \in \mathbb{R}^n$ is the generalized coordinates; $u \in \mathbb{R}^n$ is the vector of control inputs satisfying constraint Eq.(2); $\bar{u}_i > 0, i = 1, \cdots, n$ is the positive number; $D \in \mathbb{R}^{n\times n}$ is the inertia matrix function; $C \in \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal forces matrix and $G \in \mathbb{R}^n$ is the gravity force.

The robotic manipulator Eq.(1) has the following properties.

Property 1 ([9]) *There exist some positive constants d_m, d_M, k_c and k_g such that the following properties hold for all $q, z \in \mathbb{R}^n$:*

P1. $d_m \le \|D(q)\| \le d_M$.

P2. $\|C(q,z)\| \le k_c\|z\|$.

P3. Matrix $\dot{D}(q) - 2C(q,\dot{q})$ is skew-symmetric, i.e., $\dot{q}^T\big(D(q) - 2C(q,\dot{q})\big)\dot{q} = 0$.

P4. $sup_{q\in\mathbb{R}^n}\|G(q)\| \le k_g$

P5. The left-hand side of the Eq.(1) can be rewritten as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\theta = u \quad (3)$$

where $Y(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n\times p}$ is the regressor matrix of known functions, and $\theta \in \mathbb{R}^p$ is a vector of parameters.

The problem considered in this paper can be stated as:

Problem 1 *Consider the system described by Eq.(1) and Eq.(2). Let $q_d(t) \in \mathbb{R}^n$ be a given sufficiently smooth desired trajectory with its time-derivatives bounded. Suppose that the matrices $D(q), C(q,\dot{q})$ and $G(q)$ are unknown. Design the control u for (1) such that*

1) *The constraint Eq.(2) is satisfied.*
2) *All the closed-loop signals are bounded.*
3) *The tracking error $\|q(t) - q_{d(t)}\|$ converges a neighborhood of the origin which can be made arbitrarily small.*

The main difficulty of the above problem is to handle simultaneously the constraint Eq.(2) and the uncertainties of $D(q), C(q,\dot{q})$ and $G(q)$. The similar problem has been considered [10,11]. Here, a different control shall be designed to solve the problem 1.

3. CONTROL DESIGN

This section presents the computation of the adaptive control u for Eq.(1) with predetermined bound Eq.(2). For this purpose, we need the following concept of saturation function.

Definition 1 *Given a positive constant $\bar{\sigma}$, a function $\sigma: \mathbb{R} \to \mathbb{R}$ is said to be a smooth saturation function with the bound $\bar{\sigma}$, if it is smooth and satisfies (i) $\varsigma\sigma(\varsigma) > 0, \forall\, \varsigma \ne 0$; (ii) $|\sigma(\varsigma)| \le \bar{\sigma}, \forall\, \varsigma \in \mathbb{R}$; (iii) $\sigma(-\varsigma) = -\sigma(\varsigma), \forall \varsigma \in \mathbb{R}$; (iv) $\forall \bar{\varsigma} > 0, \exists k > 0$ such that $|\sigma(k\varsigma)| \ge \frac{\sigma(k\bar{\varsigma})}{\bar{\varsigma}}|\varsigma|, \forall\, |\varsigma| \le \bar{\varsigma}$.*

The above definition is similar to the definition in [9]. The additional properties (iv) is necessary for our design later. Some functions satisfying the Definition 1 include $\sigma(\varsigma) = \bar{\sigma}\tanh(\varsigma), \sigma(\varsigma) = \bar{\sigma}\varsigma/\sqrt{1+\varsigma^2}$ or $\sigma(\varsigma) = \bar{\sigma}\arctan(\varsigma)$.

To solve the tracking control problem, we define the tracking errors as

$$e = q - q_d,\ r = \dot{e} + \Lambda e \quad (4)$$

where $\Lambda \in \mathbb{R}^{n\times n}$ is a positive diagonal matrix. Taking the time derivative of Eq.(4) along Eq.(1), yields

$$D(q)\dot{r} = -C(q,\dot{q})r + u - D(q)(\ddot{q}_d - \Lambda\dot{e}) - C(q,\dot{q})(\dot{q}_d - \Lambda e) - G(q). \quad (5)$$

Denoting

$$\varpi = \dot{q}_d - \Lambda e \quad (6)$$

and using the item P5 of Property 1, we have

$$Y_\varpi(q,\dot{q},\varpi)\theta = -D(q)\dot{\varpi} - C(q,\dot{q})\varpi - G(q), \quad (7)$$

where $\theta \in \mathbb{R}^p$ contains the unknown constant parameters and $Y_\varpi(q,\dot{q},\varpi) \in \mathbb{R}^{n\times p}$ contains the known functions. The equation (5) is rewritten as

$$D(q)\dot{r} = -C(q,\dot{q})r + u(v) + Y_\varpi\theta. \quad (8)$$

Add and subtract the right-hand side of Eq. (8) by

$$Y_d(q,\dot{q}_d,\ddot{q}_d)\theta = -D(q)\ddot{q}_d - C(q,\dot{q}_d)\dot{q}_d - G(q), \quad (9)$$

where $Y_d(q,\dot{q}_d,\ddot{q}_d)$ has the same form with $Y(q,\dot{q},\ddot{q})$ in Eq.(3) instead that $\dot{q}$ and $\ddot{q}$ are replaced by $\dot{q}_d$ and $\ddot{q}_d$, respectively, then Eq.(8) becomes

$$D(q)\dot{r} = -C(q,\dot{q})r + Y_d\theta + y(q,\dot{q},q_d,\dot{q}_d) + u \quad (10)$$

where

$$y(\cdot) = D(q)(\ddot{q}_d - \Lambda\dot{e}) - D(q)\ddot{q}_d + C(q,\dot{q})(\dot{q}_d - \Lambda e) - C(q,\dot{q}_d)\dot{q}_d \quad (11)$$

The goal now is to design the control u satisfying Eq.(2) for Eq.(10) to achieve $r \to 0$ as $t \to \infty$. To this end, we need the following assumption.

Assumption 1 *Uncertain parameter θ belongs to a set*

$$\mathcal{M} = \{\theta \in \mathbb{R}^p | \underline{\theta} \le \theta \le \bar{\theta}\}, \quad (12)$$

where $\underline{\theta}, \bar{\theta} \in \mathbb{R}^p$ are two known vectors satisfying $\underline{\theta} < \bar{\theta}$.

Based on Assumption 1, we propose the control u as

$$u = -\sigma_1(\Lambda e) - \sigma_2(r) - Y_d\hat{\theta} \quad (13)$$

and the update law as

$$\dot{\hat{\theta}} = \mathrm{Proj}_{\hat{\theta}}(\tau),\ \tau = \Gamma_1 Y_d r, \hat{\theta}(0) \in \mathcal{M} \quad (14)$$

where $\sigma_1, \sigma_2 \in \mathbb{R}^n$ are the smooth vector saturation functions, $\hat{\theta}$ is an estimation of θ; $\mathrm{Proj}_{\hat{\theta}}(\cdot)$ denotes the smooth projection operator from [13], which is defined as

$$\mathrm{Proj}_{\hat{\theta}} = \begin{cases} \tau & \text{if } \mathcal{P}(\hat{\theta}) \le 0 \\ \tau & \text{if } \mathcal{P}(\hat{\theta}) \ge 0 \text{ and } \nabla_{\hat{\theta}}\mathcal{P}\tau \le 0 \\ \tau - \frac{\mathcal{P}(\hat{\theta})\nabla_{\hat{\theta}}\mathcal{P}\nabla_{\hat{\theta}}\mathcal{P}^T}{\|\nabla_{\hat{\theta}}\mathcal{P}\|^2}\tau & \text{if not} \end{cases} \quad (15)$$

here $\mathcal{P}$ is a smooth convex function defined in $\mathcal{M}$ and we use denotation $\nabla_{\hat{\theta}}\mathcal{P} = \partial\mathcal{P}/\partial\hat{\theta}$.

The smooth projection operator Eq.(15) is employed to guarantee that $\hat{\theta} \in \mathcal{M}$ for all $t \ge 0$ if $\hat{\theta}(0) \in \mathcal{M}$. The bounded control Eq.(13) only achieves the local stability of the system Eq.(10). For this reason, we will construct region of attraction for Eq.(10). To do this, let two strict positive vectors $A \in \mathbb{R}^n$ and $B \in \mathbb{R}^n$ and define the set

$$\Omega_2 = \{(e, r) \in \mathbb{R}^{2n} | |e| \le A, |r| \le B\}. \quad (16)$$

We consider the candidate Lyapunov function

$$V_1(\cdot) = \sum_{i=1}^{n} \frac{1}{\Lambda_i}\int_0^{e_i} \sigma_{1i}(\Lambda_i s_i)ds_i + \frac{1}{2}r^T D(q) + \frac{1}{2}\tilde{\theta}^T \Gamma_1^{-1}\tilde{\theta} \quad (17)$$

where σ_{1i}, Λ_i and e_i are the elements of σ_1, Λ and e, respectively, and $\tilde{\theta} = \theta - \hat{\theta}$ is the estimation error of uncertain parameters, $\Gamma_1 \in \mathbb{R}^{n\times n}$ is a positive definitive matrix. Furthermore, we need the following denotations

$$k_5 = \min_{i=1,\cdots,n} \left|\frac{\sigma_{1i}(\Lambda_i a_i)}{a_i}\right|$$
$$k_6 = \min_{i=1,\cdots,n} \left|\frac{\sigma_{2i}(b_i)}{b_i}\right| \quad (18)$$
$$k_7 = \max_{i=1,\cdots,n} \frac{\partial\sigma_{1i}(\Lambda_i e_i)}{\partial e_i}$$

here a_i and b_i are the elements of vectors A and B, respectively. We have the following theorem.

Theorem 1 *Consider manipulator described by Eq.(1) and Eq.(2) under Assumptions 1, the control u described by Eq.(13), the update law $\hat{\theta}$ described by Eq.(14). Suppose that the derivatives of the reference trajectory $q_d(t)$ are bounded by*

$$|\dot{q}_{di}| \le M_i, |\ddot{q}_{di}| \le T_i,\ i = 1, \cdots, n \quad (19)$$

where $M_i \ge 0$ and $T_i \ge 0$ satisfy

$$d_M T_i + k_c\mu M_i + \bar{g}_i < \bar{u}_i,\ i = 1, \cdots, n \quad (20)$$
$$2k_5(k_6 - \lambda_{max}(\Lambda)d_M - k_c\mu) > \lambda_{max}(\Lambda)^2(\lambda_{max(\Lambda)d_M} + 2k_c\mu)^2 \quad (21)$$

with $\mu = (\sum_1^n M_i^2)^{\frac{1}{2}}$. Then, the following statements hold

1) The constraint Eq.(2) is satisfied.

2) There exists a set $\Omega_{\tilde{\theta},\varrho}$ depending on $\tilde{\theta}$ as

$$\Omega_{\tilde{\theta},\varrho} = \{(e, r) \in \Omega_2 | \zeta\, V_1(e, r, \tilde{\theta}) \le \varrho\} \quad (22)$$

with ϱ and ζ defined as

$$\varrho = \frac{4k_5(k_6 - \lambda_{max}(\Lambda)d_M - k_c\mu)}{\lambda_{max}(\Lambda)^2} - 2(\lambda_{max}(\Lambda)d_M + 2k_c\mu)^2, \quad (23)$$
$$\zeta = max\left\{\frac{8\lambda_{max}(\Lambda)^3 k_c^2 k_7}{k_6^2}, \frac{8k_c^2}{d_m}\right\} \quad (24)$$

Then, for all $(e(0), r(0)) \in \Omega_{\tilde{\theta},\varrho}$ and $\hat{\theta}(0) \in \mathcal{M}$, the tracking error converges asymptotically to the origin, i.e.,

$$lim_{t\to\infty} \|e(t), r(t)\| = 0. \quad (25)$$

Proof: 1. Denoting $F = Y_d\hat{\theta}$, from Eq.(13) we have

$$|u_i| \le \sigma_{1i}(\Lambda_i e_i) + \sigma_{2i}(r_i) + |f_i| \quad (26)$$

where f_i is the element of F. Using Eq.(9) the term f_i satisfies

$$|f_i| \le d_M T_i + k_c\mu M_i + \bar{g}_i.$$

From Eq.(20), there exist the bounds $\bar{\sigma}_{1i}$ and $\bar{\sigma}_{2i}$ such that

$$|u_i| \le \bar{\sigma}_{1i}(\Lambda_i e_i) + \bar{\sigma}_{2i}(r_i) + d_M T_i + k_c\mu M_i + \bar{g}_i \le |\bar{u}_i|. \quad (27)$$

From Eq.(27) we conclude that the constraint Eq.(2) is satisfied.

2. Substituting Eq.(13) and Eq.(14) into Eq.(10), the dynamic equations of the closed-loop tracking error are given by

$$D(q)\dot{r} = -C(q, \dot{q})r + Y_d\tilde{\theta} - \sigma_1(\Lambda e) - \sigma_2(r) + y$$
$$\dot{\hat{\theta}} = \mathrm{Proj}_{\hat{\theta}}(\Gamma_1 Y_d r). \quad (28)$$

We shall show that $\dot{V}_1$ is non-positive. Taking time derivative of V_1 in Eq.(17) along Eq.(28), yields

$$\dot{V}_1 = \dot{e}^T\sigma_1(\Lambda e) + \frac{r^T\dot{D}(q)r}{2} - r^T C(q, \dot{q})r + r^T Y_d\tilde{\theta} - r^T\sigma_1(\Lambda e) - r^T\sigma_2(r) + r^T y - \tilde{\theta}\Gamma_1^{-1}\dot{\hat{\theta}} \quad (29)$$

Using item P3 of Property 1, we have $r^T(\dot{D}(q) - 2C(q, \dot{q})r = 0$ and substitute $\dot{e}$ from Eq.(4) into Eq.(29), we obtain

$$\dot{V}_1 = -e^T\Lambda\sigma_1(\Lambda e) - r^T\sigma_2(r) + r^T y - \tilde{\theta}^T(\Gamma_1^{-1}\mathrm{Proj}_{\hat{\theta}}(\Gamma_1 Y_d r) - Y_d r) \quad (30)$$

Using Eq.(15), we have $\tilde{\theta}^T(\Gamma_1^{-1}\mathrm{Proj}_{\hat{\theta}}(\Gamma_1 Y_d r) - Ydr \ge 0$, then, Eq.(30) becomes

$$\dot{V}_1 \le -e^T\Lambda\sigma_1(\Lambda e) - r^T\sigma_2(r) + r^T y. \quad (31)$$

The last term in Eq.(31) satisfies the following inequality

$$r^T y \le (\lambda_{\max}(\Lambda)d_M + k_c\mu)\|r\|^2 + \lambda_{\max}(\Lambda)(\lambda_{\max}(\Lambda)k_c\|e\| + k_c\|r\| + \lambda_{\max}(\Lambda)d_M + 2k_c\mu)\|e\|\|r\|, \quad (32)$$

and the following inequalities hold true for all $(e, r) \in \Omega_2$

$$-\lambda_{\min}(\Lambda)e\sigma_1(\Lambda e) \le -k_5\|e\|^2$$
$$-r^T\sigma_2(r) \le -k_6\|r\|^2, \quad (33)$$

where k_5 and k_6 are in Eq.(18). From Eq.(32) and Eq.(33), we have

$$\dot{V}_1 \le -c_1\|r\|\text{^}2 - c_{2\|e\|}^2 + \lambda_{\max}(\Lambda)(\lambda_{\max}(\Lambda)k_c\|e\| + k_c\|r + c_3)\|e\|\|r\| \quad (34)$$

with $c_1 = k_6 - \lambda_{\max}(\Lambda)d_M - k_c\mu$; $c_2 = k_5$; $c_3 = \lambda_{\max}(\Lambda)d_M + 2k_c\mu$. Then, we rewrite

$$V_1 \le -x^T Q(e, r)x \quad (35)$$

where $x = [\|e\|, \|r\|]^T$ and

$$Q(\cdot) = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \quad (36)$$

With $q_{11} = c_1$; $q_{22} = c_2$; $q_{12} = q_{21} = -\lambda_{\max}(\Lambda)(\lambda_{\max}(\Lambda)k_c\|e\| + k_c\|r\| + c_3)/2$.

Thus, $\dot{V}_1(\cdot)$ is non-positive if the $Q(e,r)$ is positive definitive. This can be satisfied if $c_1 > 0, c_2 > 0$ and

$$c_1 c_2 > \frac{\lambda_{\max}(\Lambda)^2(\lambda_{\max}(\Lambda)k_c\|e\|+k_c\|r\|+c_3)^2}{4}. \quad (37)$$

Condition $c_2 > 0$ is satisfied by defining k_5 in Eq.(18). Condition $c_1 > 0$ is satisfied by Eq.(21). The remainder is condition Eq.(37) which can be written as

$$(\lambda_{\max}(\Lambda)k_c\|e\| + k_c\|r\| + c_3)^2 \leq \frac{4c_1c_2}{b\lambda_{\max}(\Lambda)^2} \quad (38)$$

The upper bound of the left-hand side of Eq.(38) is

$$(\lambda_{\max}(\Lambda)k_c\|e\| + k_c\|r\| + c_3)^2$$
$$\leq 4\lambda_{\max}(\Lambda)^2 k_c^2\|e\|^2 + 4k_c^2\|r\|^2 + 2c_3^2 \quad (39)$$

here we use the inequality $\left(\tilde{a}+\tilde{b}\right)^2 \leq 2\tilde{a}^2 + 2\tilde{b}^2$. Denoting

$$W_1(e,r) = 4\lambda_{\max}(\Lambda)^2 k_c^2\|e\|^2 + 4k_c^2\|r\|^2. \quad (40)$$

Clearly, $W(e,r)$ is a positive definitivefunction.Then, Eq.(39) becomes

$$(\lambda_{\max}(\Lambda)k_c\|e\| + k_c\|r\| + c_3)^2 \leq W_1(e,r) + 2c_3^2. \quad (41)$$

From Eq.(38), Eq.(40) and Eq.(41), the condition Eq.(37) is rewritten as

$$W_1(e,r) < \frac{4c_1c_2}{\lambda_{\max}(\Lambda)^2} - 2c_3^2 = \varrho. \quad (42)$$

whereϱ is in Eq.(23). Let us define the set

$$\Omega_3 = \{(e,r) \in \Omega_2 | W_1(e,r) \leq \varrho\}, \quad (43)$$

then, condition Eq.(37) is ensured if $(e(t),r(t))$ stays in Ω_3 for all the time. In order to achieve this purpose, we note that

$$W_2(e,r) \leq V_1\left(e,r,\tilde{\theta}\right) \leq W_3\left(e,r,\tilde{\theta}_{\max}\right) \quad (44)$$

where

$$W_2(e,r) = \frac{\|\sigma_1(\Lambda e)\|^2}{2\lambda_{\max}(\Lambda)k_7} + \frac{d_m\|r\|^2}{2} \quad (45)$$

$$W_3\left(e,r,\tilde{\theta}_{\max}\right) = \frac{\lambda_{\max}(\Lambda)k_7}{2\lambda_{\min}(\Lambda)}\|e\|^2 + \frac{d_M}{2}\|r\|^2$$
$$+ \frac{\lambda_{\max}(\Gamma_1)\left\|\tilde{\theta}_{\max}\right\|^2}{2} \quad (46)$$

$$\tilde{\theta}_{\max} = \bar{\theta} - \underline{\theta} > 0. \quad (47)$$

Furthermore, we have

$$W_1(e,r) \leq \zeta W_2(e,r). \quad (48)$$

whereζ is in Eq.(24). Since, we have $(e,r) \in \Omega_2$, then

$$\|e\|^2 \leq \sum_{i=1}^{n} \frac{a_i^2}{\sigma_{1i}(\Lambda_i a_i)^2}|\sigma_{1i}(\Lambda_i e_i)|^2. \quad (49)$$

Let $k_8 = \max_{i=1,\cdots,n} \frac{a_i^2}{\sigma_{1i}(\Lambda_i a_i)^2}$then

$$\|e\|^2 \leq k_8\|\sigma_1(\Lambda e)\|^2 \quad (50)$$

Using Eq.(50), we rewrite $W_1(e,r)$ in Eq.(40) as

$$W_1(e,r) \leq 8\lambda_{\max}(\Lambda)^3 k_c^2 k_7 k_8 \frac{\|\sigma_1(\Lambda e)\|^2}{\lambda_{\max}(\Lambda)k_7}$$
$$+ \frac{8k_c^2}{d_m}\left(\frac{d_m}{2}\|r\|^2\right). \quad (51)$$

Let ζ as Eq.(24), then, we obtain Eq.(48).

Thus, by constructing the set $\Omega_{\tilde{\theta},\varrho}$ with using Eq.(44) and Eq.(48), we have the following relationship

$$W_1(e,r) \leq \zeta\, W_2(e,r) \leq \zeta\, W_3\left(e,r,\tilde{\theta}_{\max}\right) \leq \varrho. \quad (52)$$

Furthermore, the set $\Omega_{\tilde{\theta},\varrho}$ in (22) is a subset of

$$\Omega_4 = (e,r) \in \Omega_2 | \zeta W_2(e,r) \leq \varrho \quad (53)$$

since

$$\zeta W_3\left(e,r,\tilde{\theta}_{\max}\right) \leq \varrho \Rightarrow \zeta W_2(e,r) \leq \varrho. \quad (54)$$

Furthermore, Ω_4 is a subset ofΩ_3, since

$$\zeta W_2(e,r) \leq \varrho \Rightarrow W_1(e,r) \leq \varrho \quad (55)$$

Thus, we have

$$\Omega_{\tilde{\theta},\varrho} \subset \Omega_4 \subset \Omega_3. \quad (56)$$

The nested sets in Eq.(56) show that for any $\left(e,r,\tilde{\theta}\right) \in \Omega_{\tilde{\theta},\varrho} \times \mathcal{M}$, condition Eq.(37) holds, implying that $\dot{V}_1\left(e,r,\tilde{\theta}\right) \leq 0$, then the solution starting at $\left(e(0),r(0)\right) \in \Omega_{\tilde{\theta},\varrho}$ stays in $\Omega_{\tilde{\theta},\varrho}$ for all $\theta \in \mathcal{M}$ and $t \geq 0$, and consequently in Ω_4 as well as in Ω_3.

We proceed to show that the solution $(e(t),r(t))$ converges asymptotically to the origin. Since, for any $\tilde{\theta}(0)$and $(e,r) \in \Omega_{\tilde{\theta},\varrho}$, the matrix Q is positive definitive and

$$\dot{V}_1\left(e,r,\tilde{\theta}\right) \leq -x^TQx,\ \forall\,(e,r) \in \Omega_{\tilde{\theta},\varrho} \quad (57)$$

From Eq.(57), this shows that $(e,r) \in L_2 \cap L_\infty$ and $\tilde{\theta} \in L_\infty$ and we conclude from Eq.(13)that $u \in L_\infty$. This implies, using Eq.(1) and Property 1, that $\ddot{q} \in L_\infty$, and hence from Eq.(10) and Eq.(4), that $(\dot{e},\dot{r}) \in L_\infty$. Since, $(\dot{e},\dot{r}) \in L_\infty$ and (e,r) is uniformly continuous. Therefore, using Barbalat's Lemma [14] we conclude that $(e,r) \to 0$ as $t \to \infty$ and hence $(e,\dot{e}) \to 0$as $t \to \infty$.

Remark 1*The smooth projection operator Eq.(15) used in this paper ensures that the estimation parameter $\hat{\theta}$ always belongs to the domain $\mathcal{M}$ even when the numerical error is appeared. Since the numerical error may drive $\hat{\theta}$ outside of the domain $\mathcal{M}$. In this case, the saturation projectionoperator in [11] is no longer defined.*

4. SIMULATION RESULTS

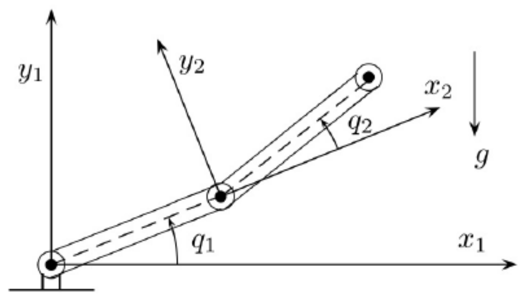

Fig.1: The Planar Elbow Manipulator with two revolute joints.

The simulation is taken from the example of a Planar Elbow Manipulator [2] with two revolute joints shown in Fig. 1. For the link $i, i = 1,2$, q_i denotes the joint angle; m_i is the mass, l_i is the length; l_{ci} denotes the distance from the previous joint to the center of mass of link i; and I_i denotes the moment of inertia about the axis coming out of the page, passing through the centre of mass of link i; The dynamic equations of the robot have the form Eq.(1) with $q = [q_1^T, q_2^T]^T$ and the matrices

$$D(q)=\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}, C(q,\dot{q})=\begin{bmatrix} h\dot{q}_2 & h\dot{q}_1+h\dot{q}_2 \\ -h\dot{q}_1 & 0 \end{bmatrix}$$

where $d_{11}=m_1 l_{c1}^2+m_2(l_1^2+l_{c2}^2+2l_1 l_{c2}\cos q_2)+I_1+I_2$; $d_{12}=d_{21}=m_2(l_{c2}^2+l_1 l_{c2}\cos q_2)+I_2$; $d_{22}=m_2 l_{c2}^2+I_2$; $h=-m_2 l_1 l_{c2}\sin q_2$ and

$$G(q)=\begin{bmatrix} (m_1 l_{c1}+m_2 l_1)g\cos q_1+m_2 l_{c2} g\cos(q\_1+q\_2) \\ m_2 l_{c2} g\cos(q\_1+q\_2) \end{bmatrix}$$

The values of the parameters in the simulation are given as follows: 0.5kg$\leq m_1\leq$3kg and 0.5kg$\leq m_2\leq$ 2 kg, l_1 =0.5m, l_{c1} = 0.25m, l_2 = 0.5m and l_{c2} = 0.25m. Furthermore, we assume that the nominal values are m_1 = 2kg, m_2 = 1.5kg. The moments of inertia are calculated by formulas $I_1=m_1 l_1^2/12$ and $I_2=m_2 l_2^2/12$. Then, we obtain $d_m=0.154$, $d_M=1.2444$, $k_c=0.4333$ and $k_g=[19.62, 4.905]^T$.

If we group the parameters as: $\theta_1=m_1 l_{c1}^2+m_{2(l_1^2+l_{c2}^2)}+I_1+I_2$; $\theta_2=m_2 l_1 l_{c2}$; $\theta_3=m_2 l_{c2}^2+I_2$; $\theta_4=m_1 l_{c1}+m_2 l_1$; $\theta_5=m_2 l_{c2}$. Then the uncertain parameter is $\theta=[\theta_1,\theta_2,\theta_3,\theta_4,\theta_5]^T\in\mathbb{R}^5$. The smooth saturation function used in the simulation is $\sigma(k\varsigma)=\bar{\sigma}tanh(k\varsigma)$ where $\bar{\sigma}$is the bound of the function $\sigma(\cdot)$.The detail of the matrix $Y_d(q,\dot{q}_d,\ddot{q}_d)$ is given in the following:

$$Y_d=\begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} & y_{15} \\ 1y_{21} & y_{22} & y_{23} & y_{24} & y_{25} \end{bmatrix} \quad (58)$$

where$y_{11}=\ddot{q}_{d1}, y_{12}=cos(q_2)(2\ddot{q}_{d1}+\ddot{q}_{d2})-$ *sinq2(qd22+2q1qd2), y13=qd2, y14=gcos(q1)* , $y_{15}=g\,cos(q_1+q_2)$, $y_{21}=0$, $y_{22}=cos(q_2)\ddot{q}_{d1}+sin(q_2)\dot{q}_{d1}^2$, $y_{23}=\ddot{q}_{d1}+\ddot{q}_{d2}$, $y_{24}=0$, $y_{25}=g\,cos(q_1+q_2)$.

The initial conditions are $q=[0{,}0]^T$rad, $\dot{q}=[0,0]^T$rad/s. The reference trajectory is given by

$$q_{d1}(t)=\frac{2\pi}{3}(1-e^{-0.5t})\text{rad},$$
$$q_{d2}(t)=\frac{\pi}{4}(1-e^{-0.5t})\text{rad} \quad (59)$$

The input saturations in (2) are

$$|u_1|\leq 30\,Nm,\ |u_2|\leq 10\,Nm \quad (60)$$

Computation of the controller Eq.(13) and Eq.(14): The bounds of uncertain parameter are:

$\underline{\theta}=[0.2082, 0.0625, 0.0417\ 0.3750\ 0.1250]^T$,
$\bar{\theta}=[1.2500,\ 0.3750, 0.2500, 2.2500, 0.7500]^T$.

The smooth convex function $\mathcal{P}(\hat{\theta})$ in Eq. (15) is constructed as

$$\mathcal{P}(\hat{\theta})=\frac{2}{\varepsilon}\left[\sum_{i=1}^{5}\left|\frac{\hat{\theta}_i-\vartheta_i}{v_i}\right|^{\ell}-1+\varepsilon\right] \quad (61)$$

where $\vartheta_i=\frac{\bar{\theta}_i+\underline{\theta}_i}{2}, v_i=\frac{\bar{\theta}_i-\underline{\theta}_i}{2}, i=1,\cdots,5$, and $0<\varepsilon<1$ and $\ell\geq 2$ are two real numbers. In the simulation, we set $\varepsilon=0.2, \ell=2$.

Following Eq. (20) and Eq. (21), we have $\mu=2.2357$, and select $\sigma_1=[4,1]^T, \sigma_2=[3,3]^T$ for Eq. (13), the matrices $\Lambda=\text{diag}(0.15{,}0.15)$ for Eq. (4), $\Gamma_1=diag(0.1{,}0.1), \hat{\theta}(0)=[0.5, 0.15, 0.1, 0.5, 0.3]^T$ for Eq. (14), $A=[3{,}3]^T$ and $B=[2{,}2]^T$ for Eq. (16). Then from Eq. (18) $k_5=0.3333, k_6=1.5, k_7=0.6$, from Eq. (23) $\varrho=11.4419$ and from Eq. (24) $\zeta=2.4227$. Thus all the conditions Eq. (20) and Eq. (21) are satisfied.

For the comparison, we carry out the simulation of the proposed controller Eq. (13) and Eq. (14), and the Model Reference Adaptive Control-like (MRAC-like) in [5], and the unbounded controller in [15]. In the results, the joint angles q_1 and q_2 are illustrated in Fig. 2. The angular velocities of two link robot have been presented in Fig. 3. The controls u_1 and u_2 have been shown in Figs. 4. We are able to see that the proposed adaptive controller does not reach the input saturations while the other two controllers violated. The estimations of uncertain parameters of three methods were illustrated in Fig. 5. Thus, It was mentioned that the estimated parameters in three methods do not converge to its real values. In addition, the uncertain parameter $\hat{\theta}$estimated by projection algorithm always belongs in region $[\underline{\theta},\bar{\theta}]$.

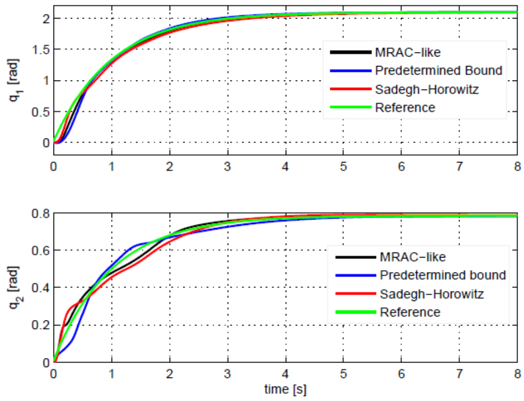

Fig. 2: Joint angles $q_1$and q_2.

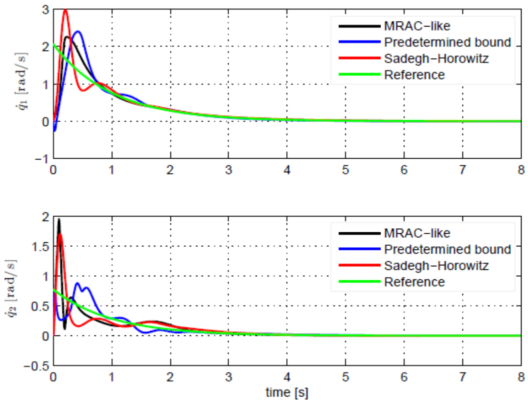

Fig. 3: Angular velocities $\dot{q}_1$and $\dot{q}_2$.

5. CONCLUSIONS

We have proposed the control schema for the uncertain robotic manipulator in the presence input saturations, that is the controller with predetermined bounds. Our controller is based on computation of the control with predetermined bounds. The simulation showed that the control objective was completed.

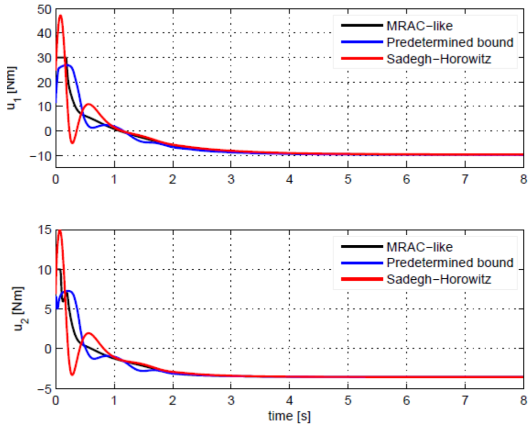

Fig. 4: Torques inputs u_1 and u_2.

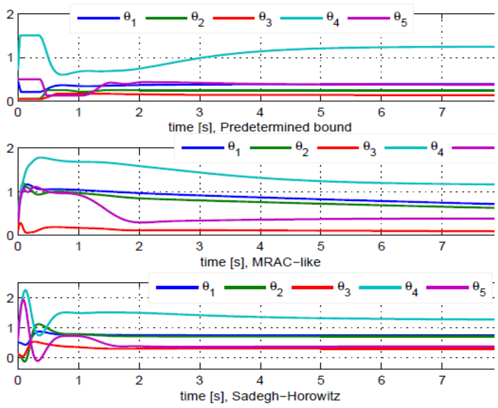

Fig. 5: Estimated parameters.

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